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A DISTORTION INPUT PARAMETER IN IMAGE DENOISING ALGORITHMS WITH WAVELETS

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Abstract: The problem of image denoising based on wavelets is considered. The paper proposes an image denoising method by imposing a distortion input parameter instead of threshold. The method has two algorithms. The first one is running off line and it is applied to the prototype of the image class and it building a specific dependency, linear or nonlinear, between the final desired distortion and the necessary probability of the details coefficients. The next algorithm, is directly applying the denoising with a threshold computed from the previous step. The threshold is estimated by using the probability density function of the details coefficients and by imposing the probability of the coefficients which will be kept. The obtained results are at the same quality level with other well known methods.

Keywords: Signal Processing, denoising, filtering, thresholding, simulation, wavelets.

1. INTRODUCTION

There are many applications where noised images are involved. Mainly, noised images are present in applications where processing noise appears, e.g. quantization or transmission processes. The classical image denoising techniques are based either on linear filtering (e.g., a low pass filter), or nonlinear (e.g., a median filter). The main difficulty of these filtering-based techniques is that they tend to blur the image. Wavelet-based denoising techniques have been recognized as powerful tools for denoising. These methods can be considered as transform-domain point processing, (Bovic, 2000). The main difficulty on image denoising based on wavelets is to choice the right threshold value in order to obtain the desired quality of the image. Obviously, there is an iterative process.

The objective of the paper is to analyze the possibility of computation of the right value of the threshold by considering an imposed distortion criterion value, as input parameter in image filtering (denoising) process. In section II, a very short review of the wavelets theory is presented. Section III describes an algorithm for the computation of the threshold based on probability of coefficients from a given level and the methodology to link the probability with an imposed distortion criterion. Section IV presents the simulation results on two input images with different types of noise by using the proposed algorithm and comparing with other well known and intensively used denoising algorithms.

The analysis is made by computer simulation.

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2. BACKGROUND

Under some physical constraints, it is possible to decompose a signal s(t) by writing the decomposition process of the signal as:

$$\begin{aligned} (1) \quad s(t) &= A_1(t) + D_1(t) \\ &= A_2(t) + D_1(t) + D_2(t) = \ldots \end{aligned}$$

where $D_i(t)$ is called the detail at level i and $A_i(t)$ is called the approximation at level *i*, (Strang and Nguyen, 1996). The decomposition is an iterative process. At each iteration the decomposition is repeated, but only for the approximation (obtained through down filtering process) resulted from the previous iteration. The algorithm can be continued until the approximation is reduced to one pixel. This is called the wavelet decomposition tree. Other details and tutorials are presented, e.g. in (Mallat, 1998), (Achim et al., 2003), or (Isar, et al., 2005).

The problem of image denoising is mainly to recover an original image, which is distorted by an additive noise, from the observation, i.e. (Bovic, 2000):

$$(2) \quad G = I + Z$$

Here, the case of additive noise is present, with the remark that there are also applications where the noise is multiplicative. In such cases, when multiplicative contamination is concerned, multiscale methods involve a preprocessing step consisting of a logarithmic transform to separate the noise from the original signal, (Achim et al., 2003). The basic methods of denoising attempt to reject noise by damping or thresholding in the wavelet domain. The estimate of the image *I* is given by:

$$(3) \quad \hat{I} = W^{-1} * T_\lambda * W * G$$

where the operators W and W^{-1} stand for the forward and inverse discrete wavelet transform, respectively, and T_λ is a wavelet-domain point wise thresholding operator with threshold λ. From the structural computation point of view, wavelet denoising involves three stages: (1) compute the DWT of the image; (2) threshold the details wavelet coefficients; (3) compute the IDWT to obtain the denoised estimate.

For images the wavelet decomposition is done as follows: in the first level of decomposition, the image is split into 4 subbands, named HH, HL, LH and LL subbands; the HH subband gives the diagonal details of the image; the HL subband gives the horizontal features while the LH subband represents the vertical structures. The LL subband is the low resolution residual consisting of low frequency components and it is this subband which is further split at higher levels of decomposition.

A thresholding operation attenuates noise energy by removing those small coefficients while maintaining signal energy by keeping these large coefficients unchanged, (Bovic, 2000).

The small wavelet coefficients are dominated by noise, while coefficients with a large absolute value carry more signal information than noise. Replacing noisy coefficients by zero and an inverse wavelet transform may lead to a reconstruction that has lower noise.

The main difficulty in signal denoising problem is to “guess” an appropriate value of the threshold. Many methods for setting the threshold have been proposed, see e.g., (Bovic, 2000). In all these cases the threshold is selected such that satisfactory noise removal is achieved.

All investigated methods have behind complex mathematics and concepts, and are not all intuitive or easy to understand.

The proposed solution for denoising is based on two algorithms. Firstly, a pre-processing algorithm is involved. The dependency between the distortions for a class of images and the necessary probability values of details coefficients is considered. Next, the threshold value, based on the probability associated with the distortion, is compute and the denoising with thresholding is activated.

The algorithm for threshold computation is based on estimated pdf (probability density function) of the details coefficients. The threshold is the value of the coefficients which have a probability greater than an imposed value.

3. DESCRIPTION OF THE THRESHOLD ALGORITHM

A method for the threshold computation is presented and it is based on the probability of details coefficients as it follows: all details coefficients that have a probability of appearance greater than an imposed value are selected to remove. At each detail level the pdf is estimated and used in the computation of the probability limits. After removing the average we may suppose, and the simulation results confirm, a symmetric pdf for details coefficients. Next, the threshold is increased until the probability of retained details is less than an imposed value, let say P. This means that we remove all coefficients with a probability greater then threshold:

$$(4) \quad P(|N(t)| > th) = P$$

The pdf is estimated by using the histogram, which is a valid and accepted approximation, (Shanmugan and Breipohl,1998). The benefit of this method is that it does not matter the type of pdf.

The method is quite close to the problem of computation of the threshold by taking into account values which are greater the noise level value, e.g. σ. This is in the context of exploiting the fact that the pdf of the details coefficients is close to Gaussian, (Isar, et al., 2005), and then it can be playing with the variance of coefficients instead of probability.

The pseudo code of the algorithm is presented now.

THRESOLD_ALGORITHM
#1: Data Inputs:
P: = Probability of keeping
w := resolution of histogram (the cell's width)
#2: Compute histogram;
th := 0; // start computing
#3: LOOP th
th := th + w;
UNTIL $P(|N(t)| > th) \geq 1 - P$
#4: Data output: th // end computing
END.

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4. DESCRIPTION OF THE PRE-PROCESSING ALGORITHM

The most time-consuming activity is to properly set the threshold value on a case-by-case basis. Moreover, an end user will prefer an algorithm based on a distortion measure value, which is in fact a filtering quality value in the context of denoising. The algorithm is designed to work for a class of images which have the same features (i.e. the same range of intensity of the pixels and the same origin, natural or artificial).

It is supposed that each class of images has a specific dependency of distortion over probability of the kept coefficients. In this work, the link between distortion and probability is established for the prototype image only, but in the case of missing him, an estimation procedure could be considered. The obtained value is supposed to be close to what is necessary for all images from the same class of images.

The dependency of the distortion of probability value is built throughout simulation, in the sense that the probability is varying from a minimum to a maximum value and then the distortion values result are stored.

When a user intends to denois an image, it will impose a desired final distortion and the necessary value of the probability is then automatically obtained from the link table (if the dependency is nonlinear) or from the dependency slope a linear case, both computed as presented in the previous paragraph.

The dependency of the distortion of probability value is built throughout simulation, in the sense that the probability is varying from a minimum to a maximum value and then the distortion values result are stored.The pseudo code of the algorithm is described below for the linear case.

PRE-PROCESSING_ALGORITHM
#1: Initializations:
// Probabilities to estimate the slope of d=f(P)
#1.1. ***P*** *= [0.01 … 0.05]; // n values*
#1.2. Read D; // the maximum value of distortion
#2: Estimate the slope of d=f(P); // for the prototype image
#2.1. FOR i = 1:n DO
#2.1.1. Compute d(i);
End;
#. 2.2. Compute slope ; // in the linear case
#3: Compute the necessary probability for the imposed distortion, D;
#4: Filter by thresholding;
END.

When a user intends to denois an image, it will impose a desired final distortion and the necessary value of the probability is then automatically obtained from the link table (if the dependency is nonlinear) or from the dependency slope in a linear case, both computed as presented in the previous paragraph.

5. SIMULATION RESULTS

The results obtained through the method mentioned above are compared with VisuShrink thresholding technique (universal threshold), where the threshold is given by $v \cdot \sqrt{2 \cdot \log m}$, where v is the noise variance and m is the number of pixels in the image, (Donoho and Johnstone,1994).

A. Case study

Two images, Lena and Barbara, for testing were considered having sizes: 512 x 512 pixels with 8-bit gray-scales. For each image, three case studies are considered: (1) image with Gaussian white noise (of mean zero and variance σ^2); (2) image with "*salt & pepper*" noise; (3) image with stain distortion type. For all cases soft thresholding functions are used, two levels of decomposition and the wavelet type is 'db5', (Daubechies,1992).

The quality parameters of images after denoising are given by: PSNR (*peak signal-to-*noise *ratio)* and MAE *(mean absolute error).*

B. Results

In the thresholding algorithm the histogram of the detail coefficients is computed to estimate the pdf. The obtained distributions are presented in Fig. 1 and it can be shown that all estimated pdfs are close to normal pdf, which is somehow expected based on the results reported by (Isar, *et al.,* 2005):

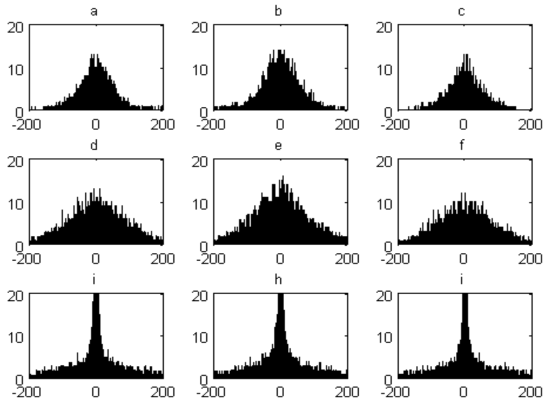

Fig.1. Histogram for detail coefficients: (a),(d),(i) horizontal coefficients when image is corrupted by Gaussian, salt & pepper and, respectively, stain noise; (b),(e),(h) vertical coefficients when image is corrupted by Gaussian, salt & pepper respectively stain noise; (c),(f),(i) the histogram for diagonal coefficients when image is corrupted by Gaussian, salt & pepper respectively stain noise.

The threshold values obtained for different values of probability are presented in the Table.

TABLE 1. THRESHOLD VALUES FOR SECOND LEVEL OF DECOMPOSITION

Study case	*P*	HL	LH	HH
Image with Gaussian white noise	0.050	78.573	83.973	73.711
	0.035	85.888	91.377	78.724
	0.020	96.538	104.13	88.142
Image with Salt & pepper noise	0.050	134.73	137.40	130.84
	0.035	143.41	147.50	142.86
	0.020	161.67	165.23	160.11
Image with stain	0.050	130.45	135.42	124.85
	0.025	147.99	153.24	143.18
	0.015	190.67	193.77	185.53

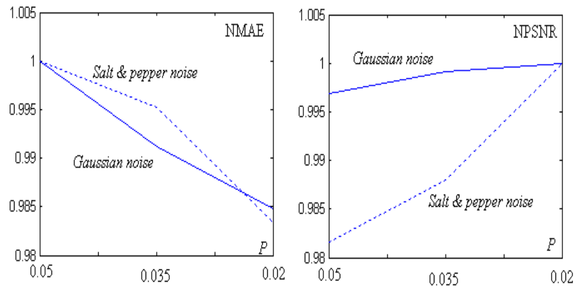

Fig.2. Evolution of MAE and PSNR, normalized to the maximum value, with probability.

It can be shown that when the probability is increasing, the threshold will decrease to low values, and this means removed of great number of detail coefficients. Therefore, the greater values for thresholding develop the better results of PSNR and MAE, i.e. PSNR increase and MAE decreases, as it might be seen in the Fig. 2.

The PSNRs of the denoised images are in [23.46, 24.6] dB interval.

Next figures illustrate an example of removing additive noise.

Fig.3. Results in denoising of 'Lena' image

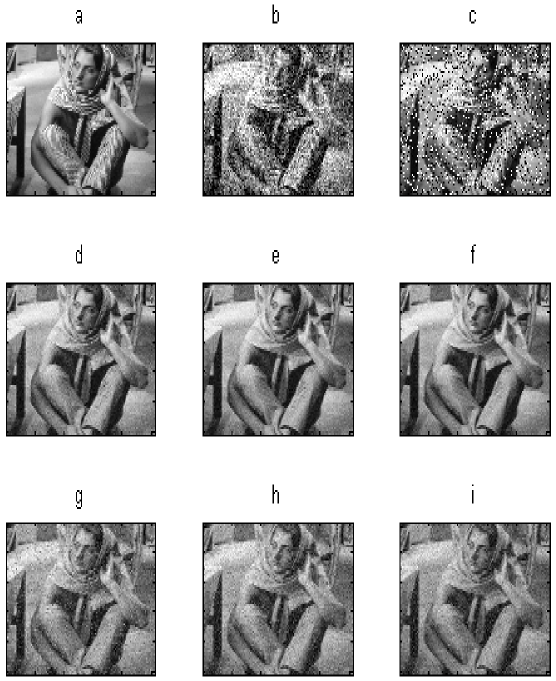

Fig.4. Results in denoising of 'Barbara' image

The images is corrupted by Gaussian noise (Fig. 3, 4-b) salt and pepper noise (Fig. 3, 4-c).

The enhanced images are obtained for P=0.05 and 0.015 for the two cases presented above, Fig. 3, 4(d), (e) respectively Fig.3, 4 (g), (h). These are compared with images enhanced through universal threshold method Fig. 3, 4 (f) respectively Fig. 3, 4 (i).

Comparing the obtained results of the denoised images, we conclude that the perceptual qualities of the enhanced images are significantly better than the noised image.

Although both methods improve the quality criteria of denoising process, the probability based function achieves better performance, i.e. higher PSNR than the VisuShrink-based method.

The proposed enhancement for denoised was also tested for a situation in which the noise represents a stain of the image Fig. 5-a, d, by considering a PSNR imposed distortion of 23 dB, which corresponds to probability of 0.015.

The results are presented in Fig. 5-b, e, and that are compared with images enhanced through universal threshold method Fig. 5-c,f.

Fig.5. Enhancement of images in which the noise represents a stain: Lena above, Barbara below

Enhanced images are better for probability based method than the universal method. Although all methods achieve better performance: higher PSNR than the VisuShrink-based method. It is to note that when the value of probability is less an improved MAE is obtained for any considered images.

6. CONCLUSION

In this paper is presented a method for denoising images based on wavelet transform, in which a distortion was imposing for the input denoising algorithm.

If is desired a better value for distortion, the value for threshold must be great, this is the result from the link between distortion and probability.

The numerically and visually achieved results are at the same level with the other values obtained in the denoising issue.

In the near future, the analysis of this algorithm method is wanted also for other types of noise and images class, and for three-dimensional wavelet transform.

The results obtained will be used also to design new denoising systems dedicated to the processing of SONAR images.

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7. REFERENCES

Al. Bovik, (2000). *Handbook of Image and Video Processing*, Academic Press.

Alexandru Isar, Sorin Moga, and Xavier Lurton (2005). *A Statistical Analysis of the 2D Discrete Wavelet Transform*, The XI th International Symposium on Applied Stochastic Models and Data Analysis – ASMDA-2005, Brest, France, May 17-20, 2005, pp.1275- 1281.

Alin Achim, Panagiotis Tsakalides, and Anastasios Bezerianos (2003). SAR Image Denoising via Bayesian Wavelet Shrinkage Based on Heavy-Tailed Modeling, *IEEE Transactions On Geoscience And Remote Sensing*, 41 (8), Pag. 1773-1784, August.

Daubechies, (1992). Ten lectures on wavelets. Philadelphia, Pa.: Society for Industrial and Applied Mathematics.

David L Donoho and I.M. Johnstone (1994). *Threshold selection for wavelet shrinkage of noisy data, Engineering Advances: New Opportunities for Biomedical Engineers*. Proceedings of the 16th Annual International Conference of the IEEE Volume, Issue, 3-6 Nov 1994 Page(s):A24 - A25 vol.1

Gilbert Strang and Truong Nguyen, (1996). Wavelets and Filter Banks", Wellesley College.

Stephane Mallat, (1998). *A wavelet tour of signals processing*, Academic Press.

Lavielle, M. (1999). *Detection of multiple changes in a sequence of dependent variables*, Stoch. Proc. and their Applications, 83, 2, pp. 79-102.

Meyer, Y., S. Roques, Eds. (1993). Progress in wavelet analysis and applications, Frontières Ed.

Shanmugan, K.S. and Breipohl, A.M., (1988). *Random Signals, Detection, Estimation and Data Analysis*, John Willey & Sons.